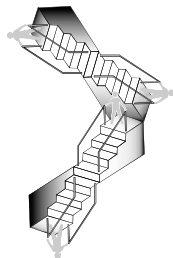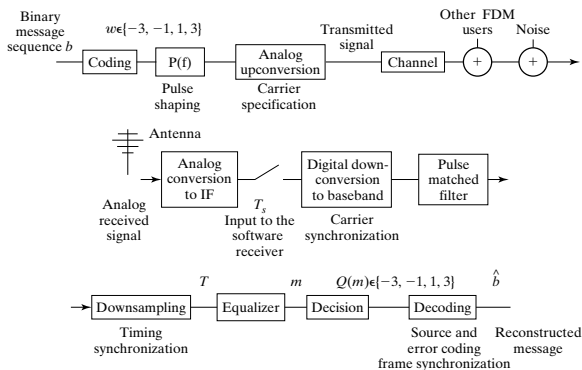# THE SIX ELEMENTS

- ⋆ PAM Communication System Building Blocks
- ⋆ Oscillators
- ⋆ Linear Filters
- ⋆ Samplers
- ⋆ Static Nonlinearities
- ⋆ Mixers
- ⋆ Adaptors

the basic components

PAM Communication System Building Blocks

- Each block in the system is composed of a number of components.
- Our approach is to identify an elemental set of building blocks from these components.
- We have selected 6 that we study individually as the basis for subsequent system design.

PAM ... Blocks (cont'd)

Our PAM communication system (and many, many others) can be built from just six types of elements:

- ▶ *Oscillators* create sine and cosine waves
- ▶ *Linear Filters* augment or diminish particular frequencies or frequency ranges from a signal
- ▶ *Static Nonlinearities* can change the frequency content of a signal, for instance multiplications, squarers, and quantizers
- ▶ *Samplers* change analog (continuous time) signals into discrete-time
- ▶ *Mixers* shift frequencies around in useful ways
- ▶ *Adaptors* track the desired values of parameters as they slowly change over time, as an augmentation to one of the other 5 elements.
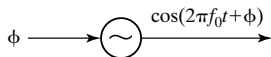
PAM ... Blocks (cont'd)

The action of an element can be thought of in terms of its effect on the time signal, or (via the Fourier Transform) in terms of its effect on the input frequency spectra.
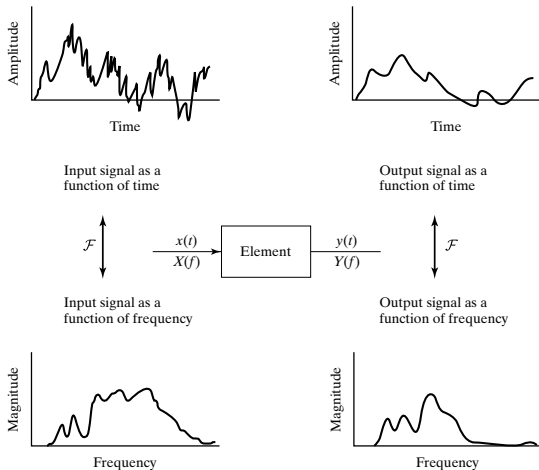
Oscillators

▶ *Symbol*:

$$\phi \longrightarrow \sim \xrightarrow{\cos(2\pi f_0 t + \phi)}$$

Often we will indicate an oscillator by simply stating the time domain description as the signal entering a system diagram branch (removing the circle with the squiggle and everything to its left).

▶ *Time Domain View*: The settings of frequency f_0 (in Hz) and phase ϕ (in radians) determine the oscillator output
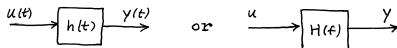
$$y(t) = \cos(2\pi f_0 t + \phi)$$

Oscillators (cont'd)

▶ *Frequency Domain View*: (Co)sinusoid in time (Fourier) transforms to impulse(s) in frequency (see (E.15) and (E.16)).

▶ *Example*: A sinusoid of frequency 10 Hz (simulated in speccos with a sample interval of 1/10th of its period) viewed (using plotspec) in time domain (top plot) and frequency domain (bottom)
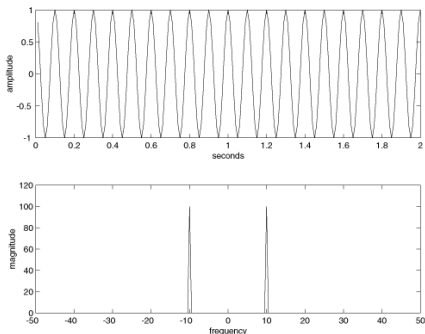
Linear Filters

► *Symbol*:

► *Time Domain View*: For a linear, time-invariant, continuous-time filter convolution of input u with impulse response h produces output y

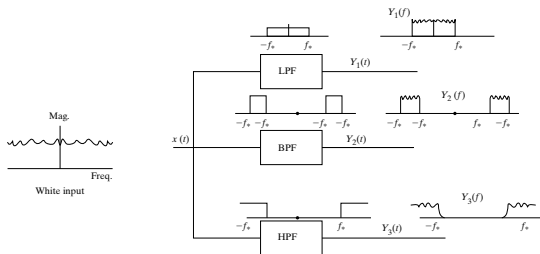$$y(t) = h(t) * u(t) = \int_{\lambda=-\infty}^{\infty} h(t-\lambda)u(\lambda)d\lambda$$

For a causal, finite-impulse-response (FIR), linear, time-invariant, discrete-time system convolution reduces to, e.g.
$y[k] = \sum_{i=0}^{2} h[i]u[k-i]$

$$= h[0]u[k] + h[1]u[k-1] + h[2]u[k-2]$$

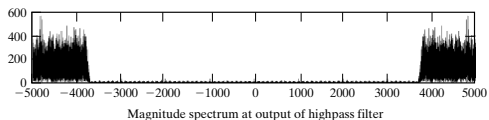where $h[k]$ is the (im)pulse response and u and y are the input and output.
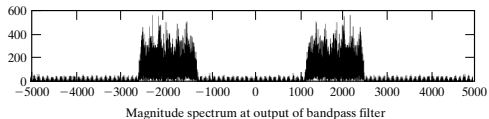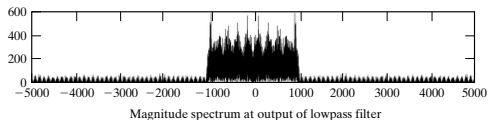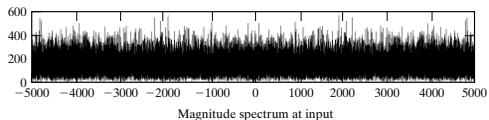
Linear Filters (cont'd)

▶ *Frequency Domain View*: Convolution in time domain is multiplication in frequency domain (see (E.37)), allowing for frequency content weighting and removal but not creation.

▶ *Example*: A 'white' signal containing all frequencies is passed through a Low Pass Filter (LPF) leaving only the low frequencies, a Band Pass Filter (BPF) leaving only the middle frequencies and a High Pass Filter (HPF) leaving only the high frequencies.
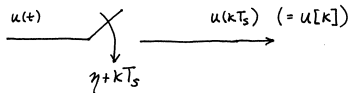
Linear Filters (cont'd)

Simulated test from `filternoise` (using `plotspec`)

Magnitude spectrum at input

Magnitude spectrum at output of lowpass filter

Magnitude spectrum at output of bandpass filter

Magnitude spectrum at output of highpass filter

Samplers

IMPULSE SAMPLER

► *Symbol*:

For a sampler with the zeroth sample ($k = 0$) synchronized with the sampler clock's designation of time zero, the offset η is zero.
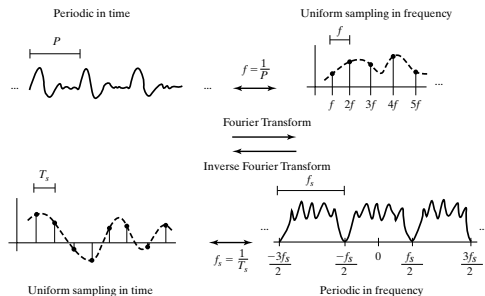
► *Time Domain View*:
- ⊙ input: $u(t)$
- ⊙ output: $u(t)|_{t=kT_s+\eta} = u(kT_s) = u[k]$

Samplers (cont'd)

▶ *Frequency Domain View*:
  ○ Sampling in time replicates analog spectrum every f_s Hz.
  ○ When plotting discrete-time signal spectrum, need only plot values over $\frac{-f_s}{2}$ to $\frac{f_s}{2}$ to be able to visualize entire spectrum.
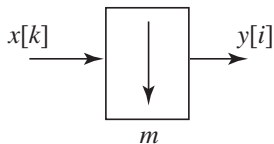
Samplers (cont'd)

▶ *Example*: 2 seconds of a square wave with period of 0.1 seconds
sampled every 0.001 seconds (sample frequency $f_s = 1000$ Hz, so
$\pm f_s/2 = \pm 500$) from specsquare using plotspec.
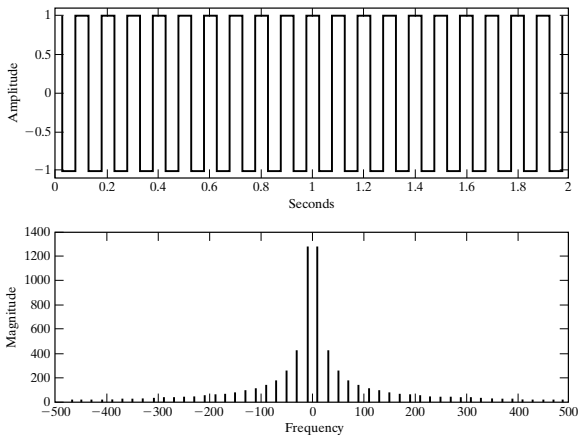
Samplers (cont'd)

DOWNSAMPLER

▶ *Symbol*:

▶ *Time Domain View*: Downsampling by m keeps only every mth sample of the previous sampled sequence.

▶ *Frequency Domain View*: Consider the analog signal that created the original sampled signal. Downsampling is like sampling the original analog signal but at a different rate. Downsampling will replicate the analog spectrum more often than the original sample rate, more likely resulting in aliasing.
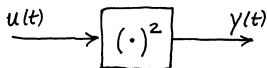
Samplers (cont'd)

▶ *Example*: For this system
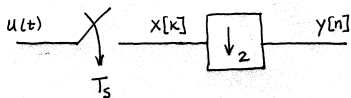
with sampling followed by downsampling, the various magnitude spectra are
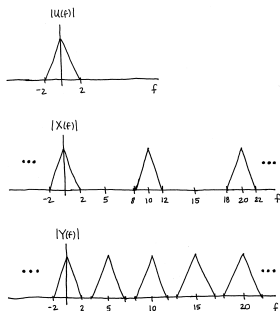
Static Nonlinearities

SQUARING

▶ *Symbol*:

$$u(t) \longrightarrow \boxed{(\cdot)^2} \longrightarrow y(t)$$

▶ *Time Domain View*: $y(t) = u^2(t)$

▶ *Frequency Domain View*: self-convolution of input spectrum; doubles width of spectrum

▶ *Example*: Consider time sinc with rectangular pulse spectrum (A.22). Spectrum of squared time sinc will be triangle pulse.

# Static Nonlinearities (cont'd)

AMPLITUDE QUANTIZATION

- *Symbol*:

- *Time Domain View*: input "converted" to nearest quantizer alphabet member
- *Frequency Domain View*: harmonics added to input spectrum
- *Example*: Square wave is a binary ± 1 quantized sinusoid. Compare spectrum plot of a 10 Hz sinusoid with preceding 10 Hz square wave $\rightarrow$ odd harmonics.
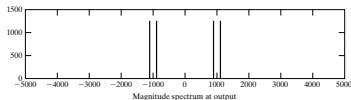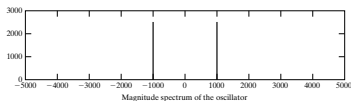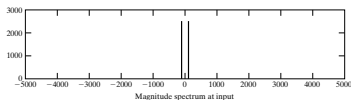
Mixers

- *Symbol*:

$u(t) \xrightarrow{\hspace{1cm}} \otimes \xrightarrow{\hspace{1cm}} y(t)$
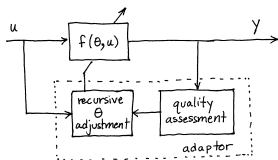
$\cos(2\pi f t + \phi)$
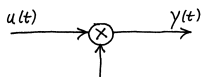
- *Time Domain View*: $y(t) = u(t)\cos(2\pi f t + \phi)$
- *Frequency Domain View*: frequency shift in input spectrum
- *Example*: input $\sim$ 100 Hz cosine; modulator $\sim$ 1000 Hz cosine

Adaptors

- *Symbol*:

- *Time Domain View*:

$$\theta[k] = \theta[k-1] + \text{correction term}$$

with correction term a function of u, y, and f.
- *Frequency Domain View*: Sequence of snapshots of spectra of input u and output y ($= f(\theta, u)$) or their ratio for successive values of θ.
- *Example*: Adaptive notch filter combatting newly materialized narrowband interferer

NEXT... We model the corruption caused by the linear channel and additive interferers.